7.1

$$
\begin{aligned}
V_{GS} &= V_{DD} = 1.8\text{ V} \\
V_{DS} &> V_{GS} - V_{TH} \text{ (in order for } M_1 \text{ to operate in saturation)} \\
V_{DS} &= V_{DD} - I_D(1\text{ k}\Omega) \\
&= V_{DD} - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{GS} - V_{TH}\right)^2 (1\text{ k}\Omega) \\
&> V_{GS} - V_{TH} \\
\frac{W}{L} &< \boxed{2.04}
\end{aligned}
$$

② To get $I_{DS} = 1\ mA$,

$$\frac{1}{2}\mu C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH})^2 = 1 \times 10^{-3}\ A.$$

$$\frac{1}{2}(200 \times 10^{-6})\left(\frac{20}{0.18}\right)_1 (V_{GS} - V_{TH})^2 = 10^{-3}$$

$$(V_{GS} - V_{TH})^2 = 0.09$$

$$V_{GS} - V_{TH} = 0.3,$$

$$\text{ie. } V_{GS} = 0.7.,$$

Since $$V_{GS} = \frac{R_2}{R_1 + R_2} \times 1.8$$

$$0.7 = \frac{R_2}{R_1 + R_2} \times 1.8.$$

$$0.7\ R_1 = .\ R_2,$$

$$\therefore \frac{R_1}{R_2} = \frac{11}{7}. \qquad \text{———— ①}$$

To get input impedance $\geq 20k$.

$$R_1 // R_2 \geq 20\ k\Omega. \qquad \text{———— ②}$$

By inspection, setting $R_1 = 55\ k\Omega$ and $R_2 = 35\ k\Omega$ will satisfy both ① and ②.

7.3

$$\begin{aligned}
V_{GS} &= V_{DD} - I_D(100\ \Omega) \\
V_{DS} &= V_{DD} - I_D(1\ \mathrm{k}\Omega + 100\ \Omega) \\
&> V_{GS} - V_{TH} \text{ (in order for } M_1 \text{ to operate in saturation)} \\
V_{DD} - I_D(1\ \mathrm{k}\Omega + 100\ \Omega) &> V_{DD} - I_D(100\ \Omega) - V_{TH} \\
I_D(1\ \mathrm{k}\Omega + 100\ \Omega) &< I_D(100\ \Omega) + V_{TH} \\
I_D(1\ \mathrm{k}\Omega) &< V_{TH} \\
I_D &< 400\ \mu\mathrm{A}
\end{aligned}$$

Since g_m increases with I_D, we should pick the maximum I_D to determine the maximum transconductance that M_1 can provide.

$$\begin{aligned}
I_{D,max} &= 400\ \mu\mathrm{A} \\
g_{m,max} &= \frac{2I_{D,max}}{V_{GS} - V_{TH}} \\
&= \frac{2I_{D,max}}{V_{DD} - I_{D,max}(100\ \Omega) - V_{TH}} \\
&= \boxed{0.588\ \mathrm{mS}}
\end{aligned}$$

④ a). $\because \quad V_{RS} = 200\,mV,$

$\therefore \quad I_{DS}\,R_S = 200\,mV$

$$I_{DS} = \frac{0.2}{100}$$

$$I_{DS} = 2\,mA.$$

For M_1 to stay in saturation,

$$V_{DS} \geqslant V_{GS} - V_{TH}.$$

$\because \quad V_{DS} = V_D - V_S$

$$= [1.8 - (2\times10^{-3})\times500] - 0.2$$

$$= 0.6,$$

$\therefore \quad V_{GS} - V_{TH} \leqslant 0.6,$

Since $\quad I_{DS} = \frac{1}{2}\,(\mu_n C_{ox})\,(\frac{W}{L})\,(V_{GS} - V_{TH})^2,$

$(\frac{W}{L})$ is min. when $(V_{GS} - V_{TH})$ is max,

$\therefore$ min $(\frac{W}{L})_1$ is when $(V_{GS} - V_{TH}) = 0.6\,V,$

$$2\times10^{-3} = \frac{1}{2}(200\times10^{-6})\,(\frac{W}{L})_1\,(0.6)^2$$

$\therefore$ min. $(\frac{W}{L})_1 \approx 56$

b) With $(V_{GS} - V_{TH}) = 0.6$,

$$V_{GS} = 1,$$

$$\therefore V_G = 1 + V_S$$

$$V_G = 1.2V,$$

i.e. $1.8 \times \dfrac{R_2}{R_1 + R_2} = 1.2V$,

$$\frac{R_2}{R_1} = 2 \quad \text{——} ①$$

Input impedance $= R_2 // R_1$,

i.e. $R_2 // R_1 \geqslant 30k\Omega$ —— ②

Set $R_1 = 50k\Omega$ and $R_2 = 100k\Omega$ will satisfy both ① & ②.

7.5

$$
\begin{aligned}
I_{D1} &= 0.5\text{ mA} \\
V_{GS} &= V_{TH} + \sqrt{\frac{2I_{D1}}{\mu_n C_{ox}\frac{W}{L}}} \\
&= 0.612\text{ V} \\
V_{GS} &= \frac{1}{10} I_{D1} R_2 \\
R_2 &= \boxed{12.243\text{ k}\Omega} \\
V_{GS} &= V_{DD} - \frac{1}{10} I_{D1} R_1 - \frac{11}{10} I_{D1} R_S \\
R_1 &= \boxed{21.557\text{ k}\Omega}
\end{aligned}
$$

7.6

$$
\begin{aligned}
I_D &= 1 \text{ mA} \\
g_m &= \frac{2I_D}{V_{GS} - V_{TH}} = \frac{1}{100} \\
V_{GS} &= 0.6 \text{ V} \\
V_{GS} &= V_{DD} - I_D R_D \\
R_D &= \boxed{1.2 \text{ k}\Omega}
\end{aligned}
$$

⑦ $I_{DS} = \frac{1}{2}(\mu_n C_{ox})(\frac{W}{L})(V_{GS} - V_{TH})^2$

$0.5 \times 10^{-3} = (100 \times 10^{-6})(\frac{50}{0.18})(V_{GS} - V_{TH})^2$

$\therefore V_{GS} = 0.534 V$

$\therefore R_2 = \frac{0.534}{0.05 \times 10^{-3}}$

$R_2 = \underline{\underline{10.68 k\Omega}}$

$\because V_{D1} = 1.8 - (1.1 \times I_{DS} \times 2 k\Omega) = 0.1 I_{DS} (R_1 + R_2)$,

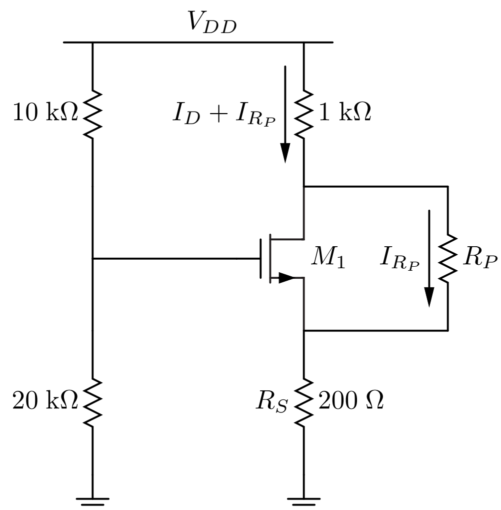
$\therefore 14 k\Omega = R_1 + 10.68 k\Omega$.

$\therefore R_1 = \underline{\underline{3320 \Omega}}$

7.8 First, let's analyze the circuit excluding R_P.

$$
\begin{aligned}
V_G &= \frac{20\ \text{k}\Omega}{10\ \text{k}\Omega + 20\ \text{k}\Omega} V_{DD} = 1.2\ \text{V} \\
V_{GS} &= V_G - I_D R_S = V_{DS} = V_{DD} - I_D (1\ \text{k}\Omega + 200\ \Omega) \\
I_D &= 600\ \mu\text{A} \\
V_{GS} &= 1.08\ \text{V} \\
\frac{W}{L} &= \frac{2 I_D}{\mu_n C_{ox} \left(V_{GS} - V_{TH}\right)^2} = 12.9758 \approx \boxed{13}
\end{aligned}
$$

Now, let's analyze the circuit with R_P.

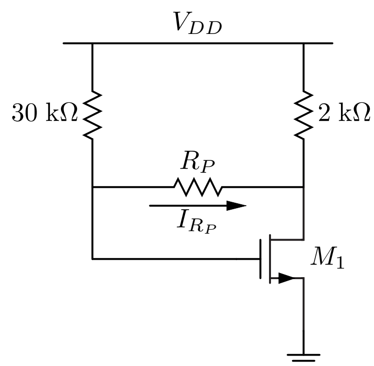


$$
\begin{aligned}
V_G &= 1.2\ \text{V} \\
I_D + I_{R_P} &= \frac{V_{DD} - V_{DS}}{1\ \text{k}\Omega + 200\ \Omega} \\
V_{GS} &= V_G - \left(I_D + I_{R_P}\right) R_S = V_{DS} + V_{TH} \\
V_G - \frac{V_{DD} - V_{DS}}{1\ \text{k}\Omega + 200\ \Omega} R_S &= V_{DS} + V_{TH} \\
V_{DS} &= 0.6\ \text{V} \\
V_{GS} &= 1\ \text{V} \\
I_D &= \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left(V_{GS} - V_{TH}\right)^2 \\
&= 467\ \mu\text{A} \\
I_D + I_{R_P} &= I_D + \frac{V_{DS}}{R_P} = \frac{V_{DD} - V_{DS}}{1\ \text{k}\Omega + 200\ \Omega} \\
R_P &= \boxed{1.126\ \text{k}\Omega}
\end{aligned}
$$

7.9 First, let's analyze the circuit excluding R_P.

$$\begin{aligned}
V_{GS} &= V_{DD} = 1.8 \text{ V} \\
V_{DS} &= V_{DD} - I_D(2 \text{ k}\Omega) = V_{GS} - 100 \text{ mV} \\
V_{DD} - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{GS} - V_{TH}\right)^2 (2 \text{ k}\Omega) &= V_{GS} - 100 \text{ mV} \\
\frac{W}{L} &= \boxed{0.255}
\end{aligned}$$

Now, let's analyze the circuit with R_P.



$$\begin{aligned}
V_{GS} &= V_{DD} - I_{R_P}(30 \text{ k}\Omega) \\
I_{R_P} &= \frac{V_{GS} - V_{DS}}{R_P} = \frac{50 \text{ mV}}{R_P} \\
V_{GS} &= V_{DD} - \left(I_D - I_{R_P}\right)(2 \text{ k}\Omega) + 50 \text{ mV} \\
V_{DD} - I_{R_P}(30 \text{ k}\Omega) &= V_{DD} - \left(\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{GS} - V_{TH}\right)^2 - I_{R_P}\right)(2 \text{ k}\Omega) + 50 \text{ mV} \\
V_{DD} - I_{R_P}(30 \text{ k}\Omega) &= V_{DD} - \left(\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{DD} - I_{R_P}(30 \text{ k}\Omega) - V_{TH}\right)^2 - I_{R_P}\right)(2 \text{ k}\Omega) + 50 \text{ mV} \\
I_{R_P} &= 1.380 \ \mu\text{A} \\
R_P &= \frac{50 \text{ mV}}{I_{R_P}} = \boxed{36.222 \text{ k}\Omega}
\end{aligned}$$

(10) For M_1,

$$I_x = \frac{1}{2}(200 \times 100^{-6})\left(\frac{W_1}{0.25}\right)(0.8-0.4)^2 \times (1 + 0.1(0.8))$$

$$10^{-3} = 0.16 \times 10^{-4}\left(\frac{W_1}{0.25}\right)(1.08)$$

$$\therefore W_1 \approx 14.5\mu$$

For M_2,

$$0.5 \times 10^{-3} = 0.16 \times 10^{-4}\left(\frac{W_2}{0.25}\right)(1.08)$$

$$\therefore W_2 = 7.25\mu$$

Output resistance $= r_o$

$$= \frac{1}{\lambda} \times \frac{1}{I_D}.$$

$$\therefore r_{o1} = \left(\frac{1}{0.1}\right)\left(\frac{1}{10^{-3}}\right)$$

$$= 10\,k\Omega$$

$$r_{o2} = \left(\frac{1}{0.1}\right)\left(\frac{1}{0.5 \times 10^{-3}}\right)$$

$$= 20\,k\Omega$$

(11) $R_{out} = \frac{1}{\lambda}\left(\frac{1}{I_D}\right)$

$$= \frac{1}{0.5\times10^{-3}\,\lambda} = 20\,k\Omega$$

$\therefore\ \lambda = 0.1\ V^{-1}$

7.12 Since we're not given V_{DS} for the transistors, let's assume $\lambda = 0$ for large-signal calculations. Let's also assume the transistors operate in saturation, since they're being used as current sources.

$$\begin{aligned}
I_X &= \frac{1}{2}\mu_n C_{ox}\frac{W_1}{L_1}\left(V_{B1} - V_{TH}\right)^2 = 0.5\text{ mA} \\
W_1 &= \boxed{3.47\ \mu\text{m}} \\
I_Y &= \frac{1}{2}\mu_n C_{ox}\frac{W_2}{L_2}\left(V_{B2} - V_{TH}\right)^2 = 0.5\text{ mA} \\
W_2 &= \boxed{1.95\ \mu\text{m}} \\
R_{out1} &= r_{o1} = \frac{1}{\lambda I_X} = 20\text{ k}\Omega \\
R_{out2} &= r_{o2} = \frac{1}{\lambda I_Y} = 20\text{ k}\Omega
\end{aligned}$$

Since $I_X = I_Y$ and λ is the same for each current source, the output resistances of the current sources are the same.

7.13 Looking into the source of M_1 we see a resistance of $\frac{1}{g_m}$. Including λ in our analysis, we have

$$\begin{aligned}\frac{1}{g_m} &= \frac{1}{\mu_p C_{ox} \frac{W}{L} \left(V_X - V_{B1} - |V_{TH}|\right)\left(1 + \lambda V_X\right)} \\ &= \boxed{372\,\Omega}\end{aligned}$$

(14) $I_X = \frac{1}{2}(100\times10^{-6})(\frac{20}{0.25})(1-1.8+0.4)^2$

$= 0.64mA$

$I_Y = \frac{1}{2}(100\times10^{-6})(2\times\frac{20}{0.25})(1-1.8+0.4)^2$

$= 1.28mA$

$\because r_o \propto \frac{1}{I}$

and $I_Y = 2I_X$

$\therefore r_{out,M_1} = 2r_{out,M_2}$

(15) $|I_{DS1}| = |I_{DS2}|$,

$$\frac{1}{2}(200\times 10^{-6})\left(\frac{10}{0.18}\right)(V_B-0.4)^2(1+0.1\times 0.9)$$
$$= \frac{1}{2}(100\times 10^{-6})(1.8-V_B-0.4)^2(1+0.1\times 0.9) \times \left(\frac{30}{0.18}\right)$$

$$2(V_B-0.4)^2 = 3(1.4-V_B)^2$$

$$\sqrt{\frac{2}{3}}(V_B-0.4) = (1.4-V_B)$$

$$1.816\,V_B = 1.7264$$

$$V_B = 0.95$$

16 a) For M_1,

$$I_{D1} = \frac{1}{2}\left(200 \times 10^{-6}\right)\left(\frac{5}{0.18}\right)(V_B - 0.4)^2 \, (1 + 0.1 \times 0.9)$$

$$\therefore V_B \approx 0.806 \, V$$

b) There are 3 regions of operation:

For $V_X < V_B - V_{TH,1}$, M_1 is in triode.

and $|I_{DS2}| > |I_{DS1}|$

For $|V_X - V_{DD}| > |V_B - V_{DD} - V_{TH2}|$, M_2 is in triode

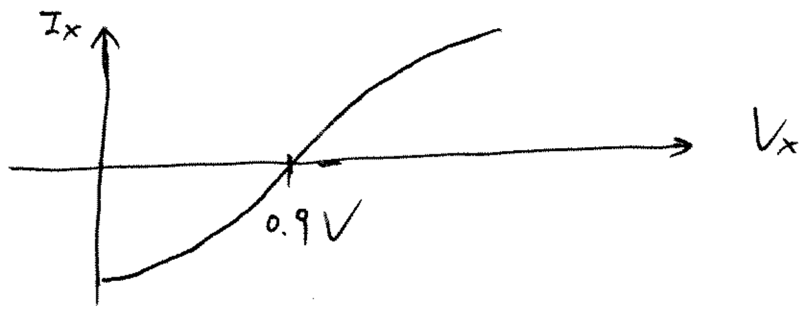
and $I_{DS1} > |I_{DS2}|$

For $V_B - V_{TH} < V_X$ and $|V_X - V_{DD}| < |V_B - V \; - V_{TH2}|$

M_1 and M_2 are in saturation.

and $I_{DS1} = |I_{DS2}| = 0.5 \, mA$ at $V_X = 0.9 \, V$

In all cases, $I_X = I_{DS1} - |I_{DS2}|$



7.17 (a) Assume M_1 is operating in saturation.

$$\begin{aligned} I_D &= 0.5 \text{ mA} \\ V_{GS} &= V_{TH} + \sqrt{\frac{2I_D}{\mu_n C_{ox} \frac{W}{L}}} \\ &= \boxed{0.573 \text{ V}} \end{aligned}$$

$$V_{DS} = V_{DD} - I_D R_D = 0.8 \text{ volt} > V_{GS} - V_{TH}, \text{ verifying that } M_1 \text{ is in saturation}$$

(b)

$$\begin{aligned} A_v &= -g_m R_D \\ &= -\frac{2I_D}{V_{GS} - V_{TH}} R_D \\ &= \boxed{-11.55} \end{aligned}$$

7.18 (a) Assume M_1 is operating in saturation.

$$I_D = 0.25 \text{ mA}$$
$$V_{GS} = V_{TH} + \sqrt{\frac{2I_D}{\mu_n C_{ox} \frac{W}{L}}}$$
$$= \boxed{0.55 \text{ V}}$$
$$V_{DS} = V_{DD} - I_D R_D = 1.3 \text{ V} > V_{GS} - V_{TH}, \text{ verifying that } M_1 \text{ is in saturation}$$

(b)

$$\begin{aligned}
V_{GS} &= 0.55 \text{ V} \\
V_{DS} &> V_{GS} - V_{TH} \text{ (to ensure } M_1 \text{ remains in saturation)} \\
V_{DD} - I_D R_D &> V_{GS} - V_{TH} \\
V_{DD} - \frac{1}{2}\mu_n C_{ox} \frac{W}{L} \left(V_{GS} - V_{TH}\right)^2 R_D &> V_{GS} - V_{TH} \\
\frac{W}{L} &< \frac{2\left(V_{DD} - V_{GS} + V_{TH}\right)}{\mu_n C_{ox} \left(V_{GS} - V_{TH}\right)^2 R_D} \\
&= 366.67 \\
&= 3.3 \frac{20}{0.18}
\end{aligned}$$

Thus, W/L can increase by a factor of $\boxed{3.3}$ while M_1 remains in saturation.

$$\begin{aligned}
A_v &= -g_m R_D \\
&= -\mu_n C_{ox} \frac{W}{L} \left(V_{GS} - V_{TH}\right) R_D \\
A_{v,max} &= -\mu_n C_{ox} \left(\frac{W}{L}\right)_{max} \left(V_{GS} - V_{TH}\right) R_D \\
&= \boxed{-22}
\end{aligned}$$

7.19

$$
\begin{aligned}
P &= V_{DD} I_D < 1\text{ mW} \\
I_D &< 556\ \mu\text{A} \\
A_v &= -g_m R_D \\
&= -\sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} R_D \\
&= -5 \\
\frac{W}{L} &< \frac{20}{0.18} \\
R_D &> \boxed{1.006\ \text{k}\Omega}
\end{aligned}
$$

7.20 (a)

$$\begin{aligned}
I_{D1} &= I_{D2} = 0.5\text{ mA} \\
A_v &= -g_{m1}\left(r_{o1} \parallel r_{o2}\right) \\
&= -\sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 I_{D1}}\left(\frac{1}{\lambda_1 I_{D1}} \parallel \frac{1}{\lambda_2 I_{D2}}\right) \\
&= -10 \\
\left(\frac{W}{L}\right)_1 &= \boxed{7.8125}
\end{aligned}$$

(b)

$$\begin{aligned}
V_{DD} - V_B &= V_{TH} + \sqrt{\frac{2\left|I_{D2}\right|}{\mu_p C_{ox}\left(\frac{W}{L}\right)_2}} \\
V_B &= \boxed{1.1\text{ V}}
\end{aligned}$$

(21) $|A_v| = g_{m1} \, (r_{o1} \parallel r_{o2})$

$g_{m1} = \sqrt{2 \times (200 \times 10^{-6}) \left(\frac{20}{0.18}\right) \times (0.001)}$

(Since V_{ds1} is not given, assume ($1 + \lambda_1 V_{DS1}$) has minimal effect on g_{m1})

$= 6.67 \ mS. \qquad (S = \Omega^{-1})$

$r_{o1} = \dfrac{1}{\lambda_1 \times I_{D1}}$

$= \dfrac{1}{0.1 \times 1mA}$

$= 10 \ k\Omega.$

$r_{o2} \simeq \infty$

$(\because \lambda_2 \ll \lambda_1)$

$\therefore |A_v| = 6.67 \times 10^{-3} \times 10^{3} \times 10$

$= 66.7$

7.22 (a) If I_{D1} and I_{D2} remain constant while W and L double, then $g_{m1} \propto \sqrt{(W/L)_1 I_{D1}}$ will not change (since it depends only on the ratio W/L), $r_{o1} \propto \frac{1}{I_{D1}}$ will not change, and $r_{o2} \propto \frac{1}{I_{D2}}$ will not change. Thus, $A_v = -g_{m1}(r_{o1} \parallel r_{o2})$ $\boxed{\text{will not change}}$.

(b) If I_{D1}, I_{D2}, W, and L double, then $g_{m1} \propto \sqrt{(W/L)_1 I_{D1}}$ will increase by a factor of $\sqrt{2}$, $r_{o1} \propto \frac{1}{I_{D1}}$ will halve, and $r_{o2} \propto \frac{1}{I_{D2}}$ will halve. This means that $r_{o1} \parallel r_{o2}$ will halve as well, meaning $A_v = -g_{m1}(r_{o1} \parallel r_{o2})$ will $\boxed{\text{decrease by a factor of } \sqrt{2}}$.

(23). To get higher voltage gain,

(a) is preferred.

For the same dimensions of transistors and same bias current,

(a) has a high "g_m" than (b),

$\because g_{m1} > g_{m2}$.

(since $\mu_n C_{ox} > \mu_p C_{ox}$)

while $(r_{o1} // r_{o2})$ is the same for both cases.

(24) $A_v = g_{m2}\,(r_{o1} \,//\, r_{o2})$

$$r_{o1} = \frac{1}{0.15 \times 0.5mA}$$

$$= 13.3\ k\Omega.$$

$$r_{o2} = \frac{1}{0.05 \times 0.5mA}$$

$$= 40\ k\Omega.$$

$\therefore r_{o1} // r_{o2} = 10\ k\Omega.$

$$\therefore 15 = \left[\sqrt{2\times(100\times10^{-6})(\tfrac{W}{L})_2\ 0.5mA}\,\right]\cdot(10\ k\Omega)$$

$$\left(\frac{W}{L}\right)_2 = 22.5$$

(25) From Eq (7.57)

$$3 = \sqrt{\frac{20/0.18}{(W/L)_2}}$$

$$\therefore (W/L)_2 \approx 12.3$$

7.26 (a)

$$
\begin{aligned}
I_{D1} &= I_{D2} = 0.5 \text{ mA} \\
V_{GS1} &= V_{TH} + \sqrt{\frac{2I_{D1}}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1}} \\
&= 0.7 \text{ V} \\
V_{DS1} &= V_{GS1} - V_{TH} \text{ (in order of } M_1 \text{ to operate at the edge of saturation)} \\
&= V_{DD} - V_{GS2} \\
V_{GS2} &= V_{DD} - V_{GS1} + V_{TH} = V_{TH} + \sqrt{\frac{2I_{D2}}{\mu_n C_{ox} \left(\frac{W}{L}\right)_2}} \\
\left(\frac{W}{L}\right)_2 &= \boxed{4.13}
\end{aligned}
$$

(b)

$$
\begin{aligned}
A_v &= -\frac{g_{m1}}{g_{m2}} \\
&= -\frac{\sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_1 I_{D1}}}{\sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_2 I_{D2}}} \\
&= -\sqrt{\frac{\left(\frac{W}{L}\right)_1}{\left(\frac{W}{L}\right)_2}} \\
&= \boxed{-3.667}
\end{aligned}
$$

(c) Since $(W/L)_1$ is fixed, we must minimize $(W/L)_2$ in order to maximize the magnitude of the gain (based on the expression derived in part (b)). If we pick the size of M_2 so that M_1 operates at the edge of saturation, then if M_2 were to be any smaller, V_{GS2} would have to be larger (given the same I_{D2}), driving M_1 into triode. Thus, $(W/L)_2$ is its smallest possible value (without driving M_1 into saturation) when M_1 is at the edge of saturation, meaning the gain is largest in magnitude with this choice of $(W/L)_2$.

7.27 (a)

$$\begin{aligned}
A_v &= -\frac{g_{m1}}{g_{m2}} \\
&= -\frac{\sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_1 I_{D1}}}{\sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_2 I_{D2}}} \\
&= -\sqrt{\frac{\left(\frac{W}{L}\right)_1}{\left(\frac{W}{L}\right)_2}} \\
&= -5 \\
\left(\frac{W}{L}\right)_1 &= \boxed{277.78}
\end{aligned}$$

(b)

$$\begin{aligned}
V_{DS1} &> V_{GS1} - V_{TH} \text{ (to ensure } M_1 \text{ is in saturation)} \\
V_{DD} - V_{GS2} &> V_{GS1} - V_{TH} \\
V_{DD} - V_{TH} - \sqrt{\frac{2I_{D2}}{\mu_n C_{ox} \left(\frac{W}{L}\right)_2}} &> \sqrt{\frac{2I_{D1}}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1}} \\
I_{D1} = I_{D2} &< \boxed{1.512 \text{ mA}}
\end{aligned}$$

7.28 For this problem, recall that looking into the drain of a transistor with a grounded gate and source we see a resistance of r_o, and looking into either terminal of a diode-connected transistor we see a resistance of $\frac{1}{g_m} \parallel r_o$.

(a)

$$A_v = \boxed{-g_{m1}\left(r_{o1} \parallel \frac{1}{g_{m2}} \parallel r_{o2}\right)}$$

(b)

$$A_v = \boxed{-g_{m1}\left(r_{o1} \parallel r_{o2} \parallel \frac{1}{g_{m3}} \parallel r_{o3}\right)}$$

(c)

$$A_v = \boxed{-g_{m1}\left(r_{o1} \parallel r_{o2} \parallel \frac{1}{g_{m3}} \parallel r_{o3}\right)}$$

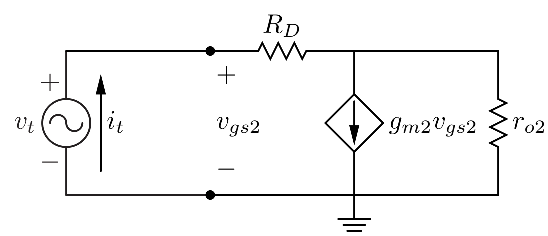
(d)

$$A_v = \boxed{-g_{m2}\left(r_{o2} \parallel r_{o1} \parallel \frac{1}{g_{m3}} \parallel r_{o3}\right)}$$

(e)

$$A_v = \boxed{-g_{m2}\left(r_{o2} \parallel r_{o1} \parallel \frac{1}{g_{m3}} \parallel r_{o3}\right)}$$

(f) Let's draw a small-signal model to find the equivalent resistance seen looking up from the output.



$$i_t = g_{m2}v_{gs2} + \frac{v_t - i_t R_D}{r_{o2}}$$

$$v_{gs2} = v_t$$

$$i_t = g_{m2}v_t + \frac{v_t - i_t R_D}{r_{o2}}$$

$$i_t\left(1 + \frac{R_D}{r_{o2}}\right) = v_t\left(g_{m2} + \frac{1}{r_{o2}}\right)$$

$$\frac{v_t}{i_t} = \frac{1 + \frac{R_D}{r_{o2}}}{g_{m2} + \frac{1}{r_{o2}}} = \frac{r_{o2} + R_D}{1 + g_{m2}r_{o2}}$$

$$A_v = \boxed{-g_{m1}\left(r_{o1} \parallel \frac{r_{o2} + R_D}{1 + g_{m2}r_{o2}}\right)}$$

7.30 (a) Assume M_1 is operating in saturation.

$$
\begin{aligned}
I_D &= 1 \text{ mA} \\
I_D R_S &= 200 \text{ mV} \\
R_S &= 200\ \Omega \\
A_v &= -\frac{R_D}{\frac{1}{g_m} + R_S} \\
&= -\frac{R_D}{\frac{1}{\sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}} + R_S} \\
&= -4 \\
\frac{W}{L} &= \boxed{1000} \\
V_{GS} &= V_{TH} + \sqrt{\frac{2I_D}{\mu_n C_{ox} \frac{W}{L}}} \\
&= 0.5 \text{ V} \\
V_{DS} &= V_{DD} - I_D R_D - I_D R_S \\
&= 0.6 \text{ V} > V_{GS} - V_{TH}, \text{ verifying that } M_1 \text{ is in saturation}
\end{aligned}
$$

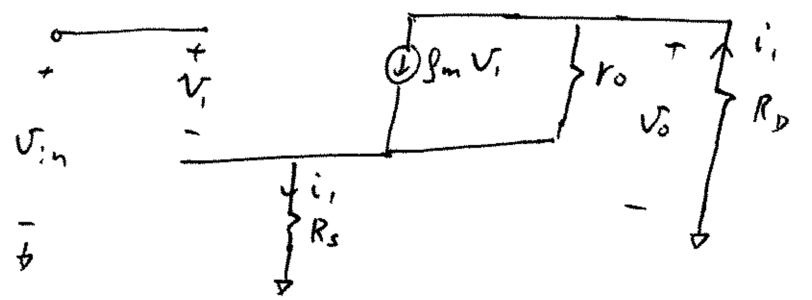
$\boxed{\text{Yes}}$, the transistor operates in saturation.

(b) Assume M_1 is operating in saturation.

$$
\begin{aligned}
\frac{W}{L} &= \frac{50}{0.18} \\
R_S &= 200\ \Omega \\
A_v &= -\frac{R_D}{\frac{1}{\sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}} + R_S} \\
&= -4 \\
R_D &= \boxed{1.179 \text{ k}\Omega} \\
V_{GS} &= V_{TH} + \sqrt{\frac{2I_D}{\mu_n C_{ox} \frac{W}{L}}} \\
&= 0.590 \text{ V} \\
V_{DS} &= V_{DD} - I_D R_D - I_D R_S \\
&= 0.421 \text{ V} > V_{GS} - V_{TH}, \text{ verifying that } M_1 \text{ is in saturation}
\end{aligned}
$$

$\boxed{\text{Yes}}$, the transistor operates in saturation.

(31) The small signal model is :



$$v_o = -i_1 R_D \quad \text{——— (1)}$$

$$i_1 = g_m v_1 + \frac{v_o - v_1}{r_o}$$

$$= \frac{(g_m r_o - 1) v_1 + v_o}{r_o}$$

$$i_1 \approx g_m v_1 + \frac{v_o}{r_o}$$

$$\therefore -\frac{v_o}{R_D} = g_m v_1 + \frac{v_o}{r_o} \quad \text{——— (2)}$$

$$v_{in} = v_1 + i_1 R_s$$

$$\therefore v_1 = v_{in} + \frac{v_o}{R_D} R_s \quad \text{——— (3)}$$

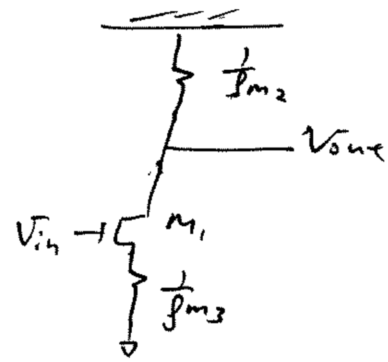
(2) combines with (3):

$$-\frac{v_o}{R_D} = g_m v_{in} + g_m v_o \frac{R_s}{R_D} + \frac{v_o}{r_o}$$

$$-v_o \left[\frac{1}{R_D} + g_m \frac{R_s}{R_D} + \frac{1}{r_o} \right] = g_m v_{in}$$

$$\therefore \text{Volt. gain} = \frac{v_o}{v_{in}} = -\left[\frac{g_m}{r_o + g_m R_s r_o + R_D}\right] (r_o R_D)$$

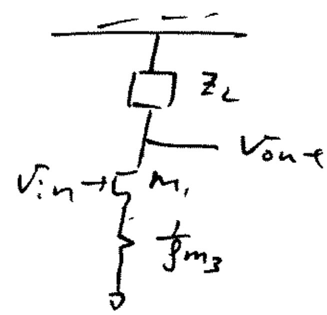
(32) a). Equivalent circuit is:



$$\therefore\ A_v = -\frac{\frac{1}{g_{m2}}}{\frac{1}{g_{m1}} + \frac{1}{g_{m3}}}$$

b) Similar to Prob. 28 (f),

Equivalent circuit is:

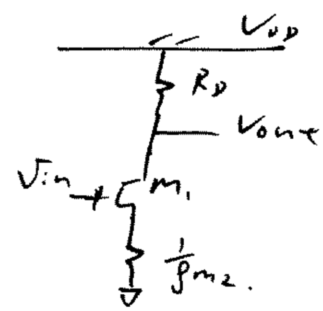


From Prob. 28 (f),

$$Z_L = \frac{1}{g_{m2}} \quad (\text{as } r_{O2} \to \infty)$$

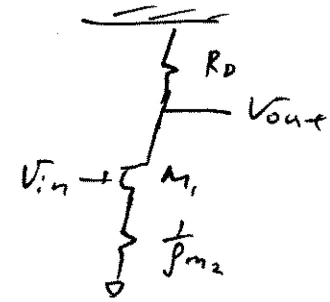
$$\therefore\ A_v = -\frac{\frac{1}{g_{m2}}}{\frac{1}{g_{m1}} + \frac{1}{g_{m3}}}$$

c). Equivalent circuit is:



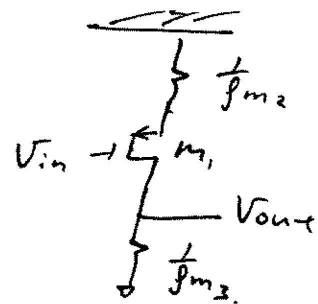
$$\therefore\ A_v = -\frac{R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}}$$

(d). Equivalent circuit is



$$A_v = -\frac{R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}}$$

(e) Equivalent circuit is



$$A_v = \frac{\frac{1}{g_{m3}}}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}}$$

(33) a) From Eq. (7.71),

$$R_{out} = (1 + g_{m1} r_{o1}) \frac{1}{g_{m2}} + r_{o1}$$

b) From Eq. (7.71),

$$R_{out} = (1 + g_{m1} r_{o1}) \frac{1}{g_{m2}} + r_{o1}$$

c) From Eq. (7.71),

$$R_{out} = (1 + g_{m2} r_{o2}) \left(r_{o1} \parallel \frac{1}{g_{m3}}\right) + r_{o2}$$

d) From Eq. (7.71),

$$R_{out} = (1 + g_{m1} r_{o1}) \left(r_{o2} \parallel \frac{1}{g_{m3}}\right) + r_{o1}$$

(34) To find $\left(\frac{W}{L}\right)$,

$$10^{-3} = \frac{1}{2}(200\times10^{-6})\left(\frac{W}{L}\right)(1-0.4)^2 \times (1+0.1\,V_{DS}),$$

where $V_{DS} = 1.8 - 1k\Omega \times 1mA$

$= 0.8V,$

$\therefore \left(\frac{W}{L}\right) \simeq 25.7$ //

Voltage gain $(A_v) = -g_{m1}(r_{o1} // R_D)$

$$g_{m1} = \sqrt{2(200\times10^{-6})(25.7)\times10^{-3}\times(1+0.1\times0.8)}$$

$= 3.33\ mS.$

$$r_{o1} = \frac{1}{0.1\times10^{-3}}$$

$= 10k\Omega.$

$\therefore A_v = (-3.33\times10^{-3})(10k\Omega // 1k\Omega)$

$= -3.03$ //

(35) with $\lambda = 0$,

$$10^{-3} = \frac{1}{2}(200 \times 10^{-6})\left(\frac{W}{L}\right)(1-0.4)^2$$

$$\therefore \left(\frac{W}{L}\right) \fallingdotseq 27.8$$

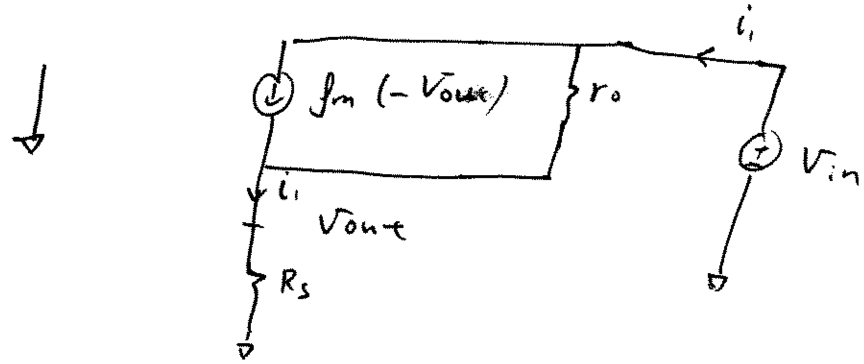
$$A_v = -g_{m1} R_D$$

$$= -\sqrt{2(200\times10^{-6})(27.8)\times10^{-3}} \times 1000$$

$$= -3.33$$

without r_o, gain increases due mainly to increase in load resistance.

(3b) The small-signal circuit is:



$$i_1 = \frac{V_{out}}{R_S} \quad \text{——— (1)}$$

$$i_1 = g_m(-V_{out}) + \frac{V_{in} - V_{out}}{r_o} \quad \text{——— (2)}$$

$$\therefore \frac{V_{out}}{R_S} = -g_m V_{out} + \frac{V_{in}}{r_o} - \frac{V_{out}}{r_o}$$

$$V_{out}\left(\frac{1}{R_S} + g_m + \frac{1}{r_o}\right) = \frac{V_{in}}{r_o}$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{r_o}\left(\frac{R_S\, r_o}{r_o + g_m r_o R_S + R_S}\right)$$

$$= \frac{R_S}{g_m r_o R_S + r_o + R_S}$$

Since $(g_m r_o R_S + r_o) > 0$, the voltage gain < 1.

This is expected: Any variation in V_{in} causes minimal change in the bias current.

$\because V_{out}$ is determined largely by the amount of bias current ($\because V_{out}$ is set by V_{BS1})

$\therefore$ There is almost no variation in V_{out}. (i.e. $\frac{V_{out}}{V_{in}} \ll 1$)

37 a) $|\text{Voltage gain}| = g_m R_D$

$$= 5$$

$$\therefore g_m = \frac{5}{500}$$

$$= 10\,mS.$$

$$= \sqrt{2(200\times10^{-6})\left(\frac{W}{L}\right)\times 10^{-3}}$$

$$\therefore \frac{W}{L} = 250$$

b) $$V_D = 1.8 - 500\times 10^{-3}$$

$$= 1.3V$$

To obtain $V_{DS} \geq V_{GS} - V_{TH} + 0.2$,

$$V_D \geq V_G - 0.2$$

$$\therefore V_G \leq 1.5$$

Also, $I_{R_1+R_2} = 0.1\times10^{-3}A.$

$$\therefore R_1 + R_2 = \frac{1.8}{0.1\times10^{-3}}$$

$$= 18\,k\Omega.$$

choose $R_2 = 15\,k\Omega$ & $R_1 = 3\,k\Omega$

c) With twice of (W/L), M_1 will go further away from triode. As (W/L) doubles, & I_{bias} is fixed by the current source, V_{GS} is forced to decrease (so M_1 will have same I_{DS}). Thus, $(V_{GS} - V_{TH})$ decreases, and V_{DS} can be allowed to drop more before M_1 goes into triode.

Gain will be increased by $\sqrt{2}$, because gain $\propto g_m$, and $g_m \propto \sqrt{W/L}$.

(38) a) $V_G = 1.8V.$

$\therefore V_{D,min} = 1.8 - 0.4$ (for M_1 stays in saturation)

$= 1.4V$

$\therefore R_{D,max} = \frac{1.4V}{1mA}$

$= 1.4k\Omega$

b) $|\text{Voltage gain}| = g_m R_D$

$= 5.$

$\therefore g_m = \frac{5}{R_D}$

$= 3.57mS.$

$= \sqrt{2 \times (200 \times 10^{-6}) (\frac{W}{L}) \times 10^{-3}}$

$\therefore (\frac{W}{L}) = 31.9$

(39) To get $R_{in} = 50\Omega$,

$$\frac{1}{g_m} = 50\Omega$$

$$\therefore g_m = 20\,mS.$$

volt gain $(A_v) = g_m R_D$

$$= 4,$$

$$\therefore R_D = \frac{4}{0.02}$$

$$R_D = 200\,\Omega$$

$$\because g_m = \sqrt{2\times(200\times10^{-6})\left(\frac{W}{L}\right)\times0.5\times10^{-3}}$$

$$\therefore \left(\frac{W}{L}\right) = 2000$$

7.42 (a)

$$\begin{aligned}
R_{out} &= R_D = 500\ \Omega \\
V_G &= V_{DD} \\
V_D &> V_G - V_{TH} \text{ (in order for } M_1 \text{ to operate in saturation)} \\
V_{DD} - I_D R_D &> V_{DD} - V_{TH} \\
I_D &< \boxed{0.8\text{ mA}}
\end{aligned}$$

(b)

$$\begin{aligned}
I_D &= 0.8\text{ mA} \\
R_{in} &= \frac{1}{g_m} \\
&= \frac{1}{\sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}} \\
&= 50\ \Omega \\
\frac{W}{L} &= \boxed{1250}
\end{aligned}$$

(c)

$$\begin{aligned}
A_v &= g_m R_D \\
g_m &= \frac{1}{50}\text{ S} \\
R_D &= 500\ \Omega \\
A_v &= \boxed{10}
\end{aligned}$$

7.43 (a)

$$\begin{aligned}
I_D &= I_1 = 1 \text{ mA} \\
V_G &= V_{DD} \\
V_D &= V_G - V_{TH} + 100 \text{ mV} \\
V_{DD} - I_D R_D &= V_G - V_{TH} + 100 \text{ mV} \\
R_D &= \boxed{300\ \Omega}
\end{aligned}$$

(b)

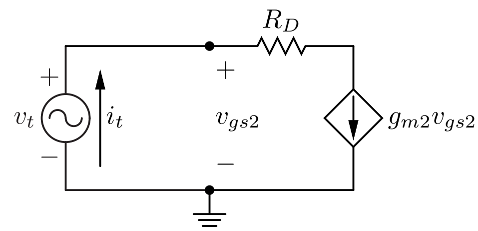
$$\begin{aligned}
R_D &= 300\ \Omega \\
A_v &= g_m R_D \\
&= \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} R_D \\
&= 5 \\
\frac{W}{L} &= \boxed{694.4}
\end{aligned}$$

7.44 For this problem, recall that looking into the drain of a transistor with a grounded gate and source we see a resistance of r_o, and looking into either terminal of a diode-connected transistor we see a resistance of $\frac{1}{g_m} \parallel r_o$.

(a) Referring to Eq. (7.109) with $R_D = \frac{1}{g_{m2}}$ and $g_m = g_{m1}$, we have

$$A_v = \boxed{\frac{\frac{1}{g_{m2}}}{\frac{1}{g_{m1}} + R_S}}$$

(b) Let's draw a small-signal model to find the equivalent resistance seen looking up from the output.



$$i_t = g_{m2}v_{gs2}$$
$$v_{gs2} = v_t$$
$$i_t = g_{m2}v_t$$
$$\frac{v_t}{i_t} = \frac{1}{g_{m2}}$$
$$A_v = \boxed{\frac{g_{m1}}{g_{m2}}}$$

(c) Referring to Eq. (7.119) with $R_D = \frac{1}{g_{m2}}$, $R_3 = R_1$, and $g_m = g_{m1}$, we have

$$A_v = \boxed{\frac{R_1 \parallel \frac{1}{g_{m1}}}{R_S + R_1 \parallel \frac{1}{g_{m1}}} \frac{g_{m1}}{g_{m2}}}$$

(d)

$$A_v = \boxed{g_{m1}\left(R_D + \frac{1}{g_{m2}} \parallel r_{o3}\right)}$$

(e)

$$A_v = \boxed{g_{m1}\left(R_D + \frac{1}{g_{m2}}\right)}$$

7.45 (a)

$$\begin{aligned}
\frac{v_X}{v_{in}} &= -g_{m1}\left(R_{D1} \parallel \frac{1}{g_{m2}}\right) \\
\frac{v_{out}}{v_X} &= g_{m2}R_{D2} \\
\frac{v_{out}}{v_{in}} &= \frac{v_X}{v_{in}}\frac{v_{out}}{v_X} \\
&= \boxed{-g_{m1}g_{m2}R_{D2}\left(R_{D1} \parallel \frac{1}{g_{m2}}\right)}
\end{aligned}$$

(b)

$$\lim_{R_{D1}\to\infty} -g_{m1}g_{m2}R_{D2}\left(R_{D1} \parallel \frac{1}{g_{m2}}\right) = \boxed{-g_{m1}R_{D2}}$$

This makes sense because the common-source stage acts as a transconductance amplifier with a transconductance of g_{m1}. The common-gate stage acts as a current buffer with a current gain of 1. Thus, the current $g_{m1}v_{in}$ flows through R_{D2}, meaning $v_{out} = -g_{m1}v_{in}R_{D2}$, so that $\frac{v_{out}}{v_{in}} = -g_{m1}R_{D2}$.

This type of amplifier (with $R_{D1} = \infty$) is known as a cascode and will be studied in detail in Chapter 9.

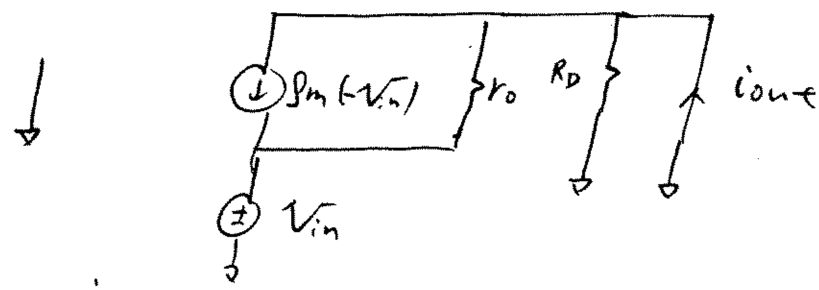
7.40

$$
\begin{aligned}
I_D &= 0.5\text{ mA} \\
R_{in} &= \frac{1}{g_m} \\
&= \frac{1}{\sqrt{2\mu_n C_{ox}\frac{W}{L}I_D}} \\
&= 50\ \Omega \\
\frac{W}{L} &= \boxed{2000} \\
V_D &> V_G - V_{TH} \text{ (in order for } M_1 \text{ to operate in saturation)} \\
V_{DD} - I_D R_D &> V_b - V_{TH} \\
R_D &< 2.4\text{ k}\Omega
\end{aligned}
$$

Since $|A_v| \propto R_D$, we need to maximize R_D in order to maximize the gain. Thus, we should pick $R_D = \boxed{2.4\text{ k}\Omega}$. This corresponds to a voltage gain of $A_v = -g_m R_D = -48$.

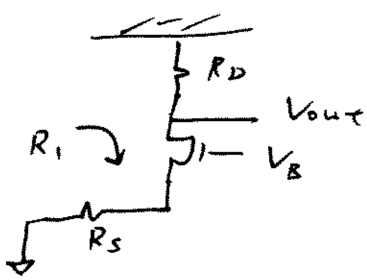
(41) Voltage gain $(A_v) = G_m R_{out}$,
where G_m and R_{out} are the transconductance and output resistance of the circuit respectively.

To find G_m:



$$G_m = \frac{i_{out}}{V_{in}} = g_m + \frac{1}{r_o}$$
$$\simeq g_m \quad (\because g_m r_o \gg 1)$$

To find R_{out}:



$$R_{out} = R_D \,//\, R_1$$
$$= R_D \,//\, [(1+g_m r_o) R_S + r_o]$$
(from Eq. (7.110))
$$\simeq R_D \,//\, (g_m r_o R_S + r_o) \quad (\because g_m r_o \gg 1)$$
$$= \frac{g_m r_o R_S R_D + r_o R_D}{R_D + g_m r_o R_S + r_o}$$

$$\therefore \text{Voltage gain} = g_m \left[\frac{g_m r_o R_D R_S + r_o R_D}{R_D + g_m r_o R_S + r_o} \right]$$

7.42 (a)

$$\begin{aligned}
R_{out} &= R_D = 500\ \Omega \\
V_G &= V_{DD} \\
V_D &> V_G - V_{TH} \text{ (in order for } M_1 \text{ to operate in saturation)} \\
V_{DD} - I_D R_D &> V_{DD} - V_{TH} \\
I_D &< \boxed{0.8\ \text{mA}}
\end{aligned}$$

(b)

$$\begin{aligned}
I_D &= 0.8\ \text{mA} \\
R_{in} &= \frac{1}{g_m} \\
&= \frac{1}{\sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}} \\
&= 50\ \Omega \\
\frac{W}{L} &= \boxed{1250}
\end{aligned}$$

(c)

$$\begin{aligned}
A_v &= g_m R_D \\
g_m &= \frac{1}{50}\ \text{S} \\
R_D &= 500\ \Omega \\
A_v &= \boxed{10}
\end{aligned}$$

7.43 (a)

$$\begin{aligned} I_D &= I_1 = 1 \text{ mA} \\ V_G &= V_{DD} \\ V_D &= V_G - V_{TH} + 100 \text{ mV} \\ V_{DD} - I_D R_D &= V_G - V_{TH} + 100 \text{ mV} \\ R_D &= \boxed{300\ \Omega} \end{aligned}$$

(b)

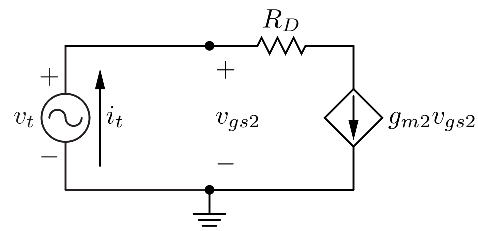
$$\begin{aligned} R_D &= 300\ \Omega \\ A_v &= g_m R_D \\ &= \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} R_D \\ &= 5 \\ \frac{W}{L} &= \boxed{694.4} \end{aligned}$$

7.44 For this problem, recall that looking into the drain of a transistor with a grounded gate and source we see a resistance of r_o, and looking into either terminal of a diode-connected transistor we see a resistance of $\frac{1}{g_m} \parallel r_o$.

(a) Referring to Eq. (7.109) with $R_D = \frac{1}{g_{m2}}$ and $g_m = g_{m1}$, we have

$$A_v = \boxed{\frac{\frac{1}{g_{m2}}}{\frac{1}{g_{m1}} + R_S}}$$

(b) Let's draw a small-signal model to find the equivalent resistance seen looking up from the output.



$$\begin{aligned}
i_t &= g_{m2} v_{gs2} \\
v_{gs2} &= v_t \\
i_t &= g_{m2} v_t \\
\frac{v_t}{i_t} &= \frac{1}{g_{m2}} \\
A_v &= \boxed{\frac{g_{m1}}{g_{m2}}}
\end{aligned}$$

(c) Referring to Eq. (7.119) with $R_D = \frac{1}{g_{m2}}$, $R_3 = R_1$, and $g_m = g_{m1}$, we have

$$A_v = \boxed{\frac{R_1 \parallel \frac{1}{g_{m1}}}{R_S + R_1 \parallel \frac{1}{g_{m1}}} \frac{g_{m1}}{g_{m2}}}$$

(d)

$$A_v = \boxed{g_{m1} \left(R_D + \frac{1}{g_{m2}} \parallel r_{o3} \right)}$$

(e)

$$A_v = \boxed{g_{m1} \left(R_D + \frac{1}{g_{m2}} \right)}$$

7.45 (a)

$$\begin{aligned}
\frac{v_X}{v_{in}} &= -g_{m1}\left(R_{D1} \parallel \frac{1}{g_{m2}}\right) \\
\frac{v_{out}}{v_X} &= g_{m2}R_{D2} \\
\frac{v_{out}}{v_{in}} &= \frac{v_X}{v_{in}}\frac{v_{out}}{v_X} \\
&= \boxed{-g_{m1}g_{m2}R_{D2}\left(R_{D1} \parallel \frac{1}{g_{m2}}\right)}
\end{aligned}$$

(b)

$$\lim_{R_{D1}\to\infty} -g_{m1}g_{m2}R_{D2}\left(R_{D1} \parallel \frac{1}{g_{m2}}\right) = \boxed{-g_{m1}R_{D2}}$$

This makes sense because the common-source stage acts as a transconductance amplifier with a transconductance of g_{m1}. The common-gate stage acts as a current buffer with a current gain of 1. Thus, the current $g_{m1}v_{in}$ flows through R_{D2}, meaning $v_{out} = -g_{m1}v_{in}R_{D2}$, so that $\frac{v_{out}}{v_{in}} = -g_{m1}R_{D2}$.

This type of amplifier (with $R_{D1} = \infty$) is known as a cascode and will be studied in detail in Chapter 9.

(46)

$$\frac{v_x}{v_{in}} = \left(R_{D1} \parallel \frac{1}{g_{m2}}\right) g_{m1}$$

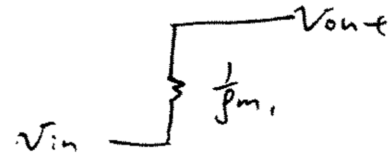
$$\frac{v_{out}}{v_x} = g_{m2} R_{D2}.$$

$$\therefore \frac{v_{out}}{v_{in}} = g_{m1} g_{m2} R_{D2} \left(R_{D1} \parallel \frac{1}{g_{m2}}\right)$$

Similar to prob. (45), voltage gain approaches that of cascode stage as R_{D1} approaches infinity. The gain is $g_{m1} R_{D2}$.

(47) with $\lambda = 0$, M_1 appears as a diode-connected device.

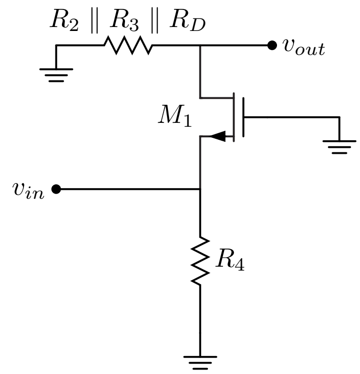
$\therefore$ the circuit becomes :



i.e. $\dfrac{V_{out}}{V_{in}} = 1$

This is not a common-gate (CG) amplifier, because the gate is not fixed. (i.e. gate is not at an "a.c. ground")

7.48 For small-signal analysis, we can short the capacitors, producing the following equivalent circuit.



$$A_v = \boxed{g_m (R_2 \parallel R_3 \parallel R_D)}$$

7.49

$$
\begin{aligned}
V_{GS} &= V_{DS} \\
V_{GS} &= V_{DD} - I_D R_S = V_{DD} - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{GS} - V_{TH}\right)^2\left(1 + \lambda V_{GS}\right) R_S \\
V_{GS} &= V_{DS} = 0.7036\ \mathrm{V} \\
I_D &= 1.096\ \mathrm{mA} \\
A_v &= \frac{r_o \parallel R_S}{\frac{1}{g_m} + r_o \parallel R_S} \\
g_m &= \sqrt{2\mu_n C_{ox}\frac{W}{L} I_D} = 6.981\ \mathrm{mS} \\
r_o &= \frac{1}{\lambda I_D} = 9.121\ \mathrm{k\Omega} \\
A_v &= \boxed{0.8628}
\end{aligned}
$$

7.50

$$
\begin{aligned}
A_v &= \frac{R_S}{\frac{1}{g_m} + R_S} \\
&= \frac{R_S}{\frac{1}{\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})} + R_S} \\
&= 0.8 \\
V_{GS} &= 0.64 \text{ V} \\
I_D &= \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left(V_{GS} - V_{TH}\right)^2 \\
&= 960 \text{ µA} \\
V_G &= V_{GS} + V_S = V_{GS} + I_D R_S \\
&= \boxed{1.12 \text{ V}}
\end{aligned}
$$

(51)

$$Av = \frac{R_s}{\frac{1}{g_m} + R_s}$$

$$= 0.8$$

$$0.8 = \frac{500}{\frac{1}{g_m} + 500}$$

$$\therefore g_m = 8 mS.$$

$$\because I_{ds} = \frac{1}{2} \beta (V_{GS} - V_t)^2,$$

where $\beta = (\frac{W}{L}) \mu_n C_{ox}$

and $g_m = \beta (V_{GS} - V_t)$.

$$\therefore I_{ds} = \frac{1}{2} g_m (V_{GS} - V_t)$$

$$= \frac{1}{2} g_m (1.8 - I_{ds}(500) - 0.4)$$

$$I_{ds} = 4 \times 10^{-3} (1.4 - 500 I_{ds})$$

$$\therefore I_{ds} = 1.87 mA.$$

$$\because g_m = \sqrt{2 (200 \times 10^{-6}) \frac{W}{L} \times 1.87 \times 10^{-3}}$$

$$\therefore \frac{W}{L} \approx 85.7$$

(52). To get $R_{out} = 100\,\Omega$,

$$\frac{1}{g_m} = 100$$

$$\therefore g_m = 10\,mS.$$

$$\because \quad I_{ds} = \frac{1}{2}\beta (V_{GS} - V_{TH})^2.$$

where $\beta = \mu_n C_{ox} \frac{W}{L}$.

and $g_m = \beta (V_{GS} - V_{TH})$

$$\therefore I_{ds} = \frac{1}{2} g_m (V_{GS} - V_{TH})$$

$$= \frac{1}{2}(10 \times 10^{-3})(0.9 - 0.4)$$

$$\therefore I_{ds} = 2.5\,mA.$$

$$\because g_m = \sqrt{2 \times (200 \times 10^{-6}) (\tfrac{W}{L}) (2.5 \times 10^{-3})}$$

$$\therefore \left(\frac{W}{L}\right) = 100$$

(53). To get $R_{out} = 50\Omega$,

$$\frac{1}{g_m} = 50\Omega$$

$$\therefore g_m = 20\,mS.$$

$$\text{Power } (P) = 1.8 \times I_{DS}$$

$$= 2 \times 10^{-3}\,W.$$

$$\therefore I_{DS} = 1.11\,mA.$$

$$\because g_m = \sqrt{2 \times (200 \times 10^{-6})\left(\frac{W}{L}\right)(1.11mA)}$$

$$\therefore \frac{W}{L} = 900$$

(54)

$$A_v = \frac{R_L}{\frac{1}{g_m} + R_L}$$

$$\therefore 0.8 = \frac{50}{\frac{1}{g_m} + 50}$$

$$g_m = 80\ mS$$

$$\text{Power } (P) = 1.8 \times I_{DS}$$

$$= 3\ mW$$

$$\therefore I_{DS} = 1.67\ mA$$

$$g_m = \sqrt{2 \times (200 \times 10^{-6}) \left(\frac{W}{L}\right) (1.67 \times 10^{-3})}$$

$$\therefore \left(\frac{W}{L}\right) = 9600$$

7.55 For this problem, recall that looking into the drain of a transistor with a grounded gate and source we see a resistance of r_o, and looking into either terminal of a diode-connected transistor we see a resistance of $\frac{1}{g_m} \parallel r_o$.

(a)

$$A_v = \boxed{\frac{r_{o1} \parallel (R_S + r_{o2})}{\frac{1}{g_{m1}} + r_{o1} \parallel (R_S + r_{o2})}}$$

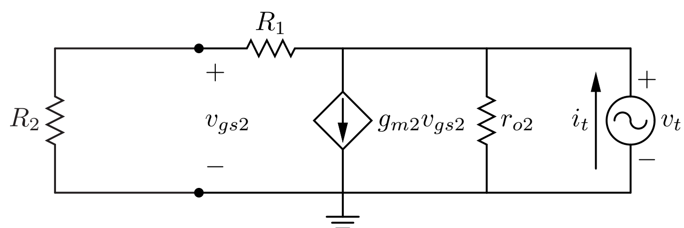
(b) Looking down from the output we see an equivalent resistance of $r_{o2} + (1 + g_{m2} r_{o2}) R_S$ by Eq. (7.110).

$$A_v = \boxed{\frac{r_{o1} \parallel [r_{o2} + (1 + g_{m2} r_{o2}) R_S]}{\frac{1}{g_{m1}} + r_{o1} \parallel [r_{o2} + (1 + g_{m2} r_{o2}) R_S]}}$$

(c)

$$A_v = \boxed{\frac{r_{o1} \parallel \frac{1}{g_{m2}}}{\frac{1}{g_{m1}} + r_{o1} \parallel \frac{1}{g_{m2}}}}$$

(d) Let's draw a small-signal model to find the equivalent resistance seen looking down from the output.



$$i_t = \frac{v_t}{R_1 + R_2} + g_{m2} v_{gs2} + \frac{v_t}{r_{o2}}$$

$$v_{gs2} = \frac{R_2}{R_1 + R_2} v_t$$

$$i_t = \frac{v_t}{R_1 + R_2} + g_{m2} \frac{R_2}{R_1 + R_2} v_t + \frac{v_t}{r_{o2}}$$

$$i_t = v_t \left(\frac{1}{R_1 + R_2} + \frac{g_{m2} R_2}{R_1 + R_2} + \frac{1}{r_{o2}} \right)$$

$$\frac{v_t}{i_t} = (R_1 + R_2) \parallel \left(\frac{R_1 + R_2}{g_{m2} R_2} \right) \parallel r_{o2}$$

$$A_v = \boxed{\frac{r_{o1} \parallel (R_1 + R_2) \parallel \left(\frac{R_1 + R_2}{g_{m2} R_2} \right) \parallel r_{o2}}{\frac{1}{g_{m1}} + r_{o1} \parallel (R_1 + R_2) \parallel \left(\frac{R_1 + R_2}{g_{m2} R_2} \right) \parallel r_{o2}}}$$

(e)

$$A_v = \boxed{\frac{r_{o2} \parallel r_{o3} \parallel \frac{1}{g_{m1}}}{\frac{1}{g_{m2}} + r_{o2} \parallel r_{o3} \parallel \frac{1}{g_{m1}}}}$$

(f) Looking up from the output we see an equivalent resistance of $r_{o2} + (1 + g_{m2} r_{o2}) r_{o3}$ by Eq. (7.110).

$$A_v = \boxed{\frac{r_{o1} \parallel [r_{o2} + (1 + g_{m2} r_{o2}) r_{o3}]}{\frac{1}{g_{m1}} + r_{o1} \parallel [r_{o2} + (1 + g_{m2} r_{o2}) r_{o3}]}}$$

(5 6)

$$\frac{v_x}{v_{in}} = \frac{\frac{1}{g_{m2}}}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}}.$$

$$\frac{V_{out}}{v_x} = g_{m2} R_D$$

$$\therefore \frac{V_{out}}{V_{in}} = \frac{R_D}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}}$$

b) if $g_{m1} = g_{m2}$,

$$\frac{V_{out}}{V_{in}} = \frac{g_{m1} R_D}{2}.$$

(57) $\because R_{out} = 1k\Omega.$

$\therefore R_D = 1k\Omega.$

$\because A_v = 5$

$= g_{m1} R_D$

$\therefore g_{m1} (1000) = 5$

$g_{m1} = 5mS.$

$\because M_1$ is 00 mV away from triode,

$V_D = (V_G - V_{TH}) + 0.1.$

$V_D = (1.8 - 0.4) + 0.1$

$V_D = 1.5V$

$\therefore I_{DS} = \dfrac{1.8 - 1.5}{R_D} = \dfrac{0.3}{R_D}$

$= 0.3mA$

$\because g_{m1} = \sqrt{2 \times (200 \times 10^{-6}) (\frac{W}{L}) I_{DS1}}$

$\therefore (\frac{W}{L}) \approx 208$

$\therefore R_D = 1k\Omega, R_G = 10k\Omega, (\frac{W}{L}) = 208$

7.58

$$\begin{aligned}
P &= V_{DD} I_D = 2 \text{ mW} \\
I_D &= 1.11 \text{ mA} \\
R_D I_D &= 1 \text{ V} \\
R_D &= 900\ \Omega \\
A_v &= -g_m R_D \\
&= -\sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D} R_D \\
&= -5 \\
\frac{W}{L} &= \boxed{69.44}
\end{aligned}$$

(58) $|A_v| = g_m R_L$.

$\therefore$ To achieve maximum gain, use maximum R_L

i.e. set $R_D = 500\,\Omega$.

For maximum g_m, use maximum I_{DS}.

(while keeping M_1 in saturation),

i.e. $V_D \geq V_G - V_{TH}$

$$1.8 - (I_{DS})(500) \geq 1.8 - 0.4 ,$$

$$\therefore \quad I_{DS} \leq \frac{0.4}{500}$$

$$I_{DS,max} = 0.8\,mA.$$

<u>Note</u>: Setting a large R_D in this case would force $I_{DS,max}$ to be lower (in order to keep M_1 in saturation).

But since $A_v \propto R_D$, while $A_v \propto \sqrt{I_{DS}}$, sacrificing I_{DS} to get higher R_D would yield a higher gain.

7.60 Let's let R_1 and R_2 consume exactly 5 % of the power budget (which means the branch containing R_D, M_1, and R_S will consume 95 % of the power budget). Let's also assume $V_{ov} = V_{GS} - V_{TH} = 300$ mV exactly.

$$\begin{aligned}
I_D V_{DD} &= 0.95(2\text{ mW}) \\
I_D &= 1.056\text{ mA} \\
I_D R_S &= 200\text{ mV} \\
R_S &= \boxed{189.5\ \Omega} \\
V_{ov} &= V_{GS} - V_{TH} = 300\text{ mV} \\
I_D &= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{ov}^2 \\
\frac{W}{L} &= \boxed{117.3} \\
A_v &= -\frac{R_D}{\frac{1}{g_m} + R_S} \\
&= -\frac{R_D}{\frac{1}{\sqrt{2\mu_n C_{ox}\frac{W}{L}I_D}} + R_S} \\
&= -4 \\
R_D &= \boxed{1.326\text{ k}\Omega} \\
\frac{V_{DD}^2}{R_1 + R_2} &= 0.05(2\text{ mW}) \\
R_1 + R_2 &= \frac{V_{DD}^2}{0.1\text{ mW}} \\
V_G &= V_{GS} + I_D R_S = V_{ov} + V_{TH} + I_D R_S = 0.9\text{ V} \\
V_G &= \frac{R_2}{R_1 + R_2}V_{DD} \\
&= \frac{R_2}{\frac{V_{DD}^2}{0.1\text{ mW}}} = 0.9\text{ V} \\
R_2 &= \boxed{29.16\text{ k}\Omega} \\
R_1 &= \boxed{3.24\text{ k}\Omega}
\end{aligned}$$

7.61 Let's let R_1 and R_2 consume exactly 5 % of the power budget (which means the branch containing R_D, M_1, and R_S will consume 95 % of the power budget).

$$
\begin{aligned}
R_D &= 200\ \Omega \\
I_D V_{DD} &= 0.95(6\text{ mW}) \\
I_D &= 3.167\text{ mA} \\
I_D R_S &= V_{ov} = V_{GS} - V_{TH} \\
R_S &= \frac{V_{ov}}{I_D} \\
g_m &= \frac{2I_D}{V_{ov}} \\
A_v &= -\frac{R_D}{\frac{1}{g_m} + R_S} \\
&= -\frac{R_D}{\frac{V_{ov}}{2I_D} + \frac{V_{ov}}{I_D}} \\
&= -5 \\
V_{ov} &= 84.44\text{ mV} \\
R_S &= \boxed{26.67\ \Omega} \\
\frac{W}{L} &= \frac{2I_D}{\mu_n C_{ox} V_{ov}^2} = \boxed{4441} \\
\frac{V_{DD}^2}{R_1 + R_2} &= 0.05(6\text{ mW}) \\
R_1 + R_2 &= \frac{V_{DD}^2}{0.3\text{ mW}} \\
V_G &= V_{GS} + I_D R_S = V_{ov} + V_{TH} + I_D R_S = 0.5689\text{ V} \\
V_G &= \frac{R_2}{R_1 + R_2} V_{DD} \\
&= \frac{R_2}{\frac{V_{DD}^2}{0.3\text{ mW}}} = 0.5689\text{ V} \\
R_2 &= \boxed{6.144\text{ k}\Omega} \\
R_1 &= \boxed{4.656\text{ k}\Omega}
\end{aligned}
$$

7.62

$$
\begin{aligned}
R_{in} &= R_1 = \boxed{20\ \text{k}\Omega} \\
P &= V_{DD} I_D = 2\ \text{mW} \\
I_D &= 1.11\ \text{mA} \\
V_{DS} &= V_{GS} - V_{TH} + 200\ \text{mV} \\
V_{DD} - I_D R_D &= V_{DD} - V_{TH} + 200\ \text{mV} \\
R_D &= 180\ \Omega \\
A_v &= -g_m R_D \\
&= -\sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} R_D \\
&= -6 \\
\frac{W}{L} &= \boxed{2500} \\
V_{GS} &= V_{TH} + \sqrt{\frac{2I_D}{\mu_n C_{ox} \frac{W}{L}}} \\
&= 0.467\ \text{V} \\
V_{GS} &= V_{DD} - I_D R_S \\
R_S &= \boxed{1.2\ \text{k}\Omega} \\
\frac{1}{2\pi f C_1} &\ll R_1 \\
\frac{1}{2\pi f C_1} &= \frac{1}{10} R_1 \\
f &= 1\ \text{MHz} \\
C_1 &= \boxed{79.6\ \text{pF}} \\
\frac{1}{2\pi f C_S} \parallel R_S &\ll \frac{1}{g_m} \\
\frac{1}{2\pi f C_S} &= \frac{1}{10} \frac{1}{g_m} \\
g_m &= \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} = 33.33\ \text{mS} \\
C_S &= \boxed{52.9\ \text{nF}}
\end{aligned}
$$

(63). Power $(P) = 2mW$.

$\therefore I_{DS1} = |I_{DS2}| = \frac{2mW}{1.8V} = 1.11mA$.

$r_{o1} = r_{o2} = \frac{1}{\lambda I_{DS}}$

$= \frac{1}{0.1 \times 1.11 \times 10^{-3}}$

$= 9000 \Omega$.

Gain $(A_v) = g_{m1} (r_{o1} // r_{o2}) = 20$.

$g_{m1} \left(\frac{9000}{2} \right) = 20$.

$\therefore g_{m1} = 4.44 \ mS$.

Set V_{DS1} (i.e. V_{out}) $= 1.2V$

(which is $< 1.5V$)

$\therefore V_{IN} = V_{GS1} \leq 1.2 + V_{TH}$

(for M_1 to stay in saturation)

Set $V_{GS1} = 1.2V$

$\because g_{m1} = \mu_n C_{ox} \left(\frac{W}{L} \right)_1 (V_{GS1} - V_{TH})$

$\left(\frac{W}{L} \right)_1 = 27.75$

For M_2, $\because M_2$ must be in saturation for $V_{out} \leq 1.5V$.

$\therefore V_{DD} - V_B \leq V_{DD} - 1.5V + V_{TH}$.

$\therefore V_B \geq 1.1V$

Set $V_B = 1.2V$

$$|I_{DS2}| = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_2 (|V_{GS2}| - V_{TH})^2$$
$$(1 + \lambda |V_{DS2}|)$$

$$1.11 \times 10^{-3} = \frac{1}{2} \times 100 \times 10^{-6} \left(\frac{W}{L}\right)_2 (0.6 - 0.2)^2$$
$$(1 + 0.1 \times (1.8 - 1.5))$$

(assuming $V_{out} = 1.5V$)

$$\therefore \left(\frac{W}{L}\right)_2 \approx 135$$

$$\therefore \left(\frac{W}{L}\right)_1 = 27.75 \qquad \left(\frac{W}{L}\right)_2 = 135$$

$V_{IN} = 1.2 \qquad V_b = 1.1$

$I_{DS1} = I_{DS2} = 1.11mA$

7.64 (a)

$$A_v = \boxed{-g_{m1}\left(r_{o1} \parallel R_G \parallel r_{o2}\right)}$$

(b)

$$\begin{aligned}
P &= V_{DD} I_{D1} = 3 \text{ mW} \\
I_{D1} &= |I_{D2}| = 1.67 \text{ mA} \\
|V_{GS2}| &= |V_{DS2}| = V_{DS} = \frac{V_{DD}}{2} \\
|I_{D2}| &= \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 \left(|V_{GS2}| - |V_{TH}|\right)^2 \left(1 + \lambda_p |V_{DS2}|\right) \\
\left(\frac{W}{L}\right)_2 &= \boxed{113} \\
A_v &= -g_{m1}\left(r_{o1} \parallel R_G \parallel r_{o2}\right) \\
R_G &= 10\left(r_{o1} \parallel r_{o2}\right) \\
r_{o1} &= \frac{1}{\lambda_n I_{D1}} = 6 \text{ k}\Omega \\
r_{o2} &= \frac{1}{\lambda_p |I_{D2}|} = 3 \text{ k}\Omega \\
R_G &= 10\left(r_{o1} \parallel r_{o2}\right) = \boxed{20 \text{ k}\Omega} \\
A_v &= -\sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 I_{D1}}\left(r_{o1} \parallel R_G \parallel r_{o2}\right) \\
&= -15 \\
\left(\frac{W}{L}\right)_1 &= \boxed{102.1} \\
V_{IN} &= V_{GS1} = V_{TH} + \sqrt{\frac{2I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left(1 + \lambda_n V_{DS1}\right)}} \\
&= \boxed{0.787 \text{ V}}
\end{aligned}$$

6.5 a) Impedance looking into drain of M_2

$$= (1 + g_{m2} r_{o2}) R_S + r_{o2}$$

$$= 10 r_{o1}$$

Assume $g_{m2} r_{o2} \gg 1$.

$\therefore g_{m2} r_{o2} R_S + r_{o2} \approx 10 r_{o1}$

$\because r_{o1} = r_{o2}$ $(\lambda_1 = \lambda_2$ and $I_{DS1} = |I_{DS2}|)$

$\therefore g_{m2} R_S + 1 = 10$

$g_{m2} R_S = 9$ ——— (1)

Given $V_B = 1V$,

set $|V_{GS2}| = 0.6V$. (i.e. $V_{GS2} - V_{TH} = 0.2V$)

$\therefore V_{S2} = 1.6V$.

$\therefore V_{R_S} = 1.8V - 1.6V = 0.2V$

$\because$ Power $= 2mW$

$I_{DS1} = |I_{DS2}| = \frac{2mW}{1.8V} = 1.11mA$.

$\therefore R_S = \frac{V_{R_S}}{1.11 \times 10^{-3}} \approx 180\Omega$

From (1), $g_{m2} = \frac{9}{180} = 50 mS$.

$\because g_{m2} = \left(\frac{W}{L}\right)_2 (100 \times 10^{-6}) (V_{GS2} - V_{TH})$

$\therefore \left(\frac{W}{L}\right)_2 = 2500$

b). $|\text{gain } (A_v)| = g_{m1} (r_{o1} \,//\, 10 r_{o1})$

$$30 = g_{m1} (0.909\, r_{o1})$$

$$r_{o1} = \frac{1}{0.1 \times 1.11 \times 10^{-3}}$$

$$= 9009\,\Omega$$

$$\therefore g_{m1} = 3.66 \text{ mS.}$$

$$\because g_{m1} = \sqrt{2 (\mu_n C_{ox}) \left(\frac{W}{L}\right)_1 \times I_{DS1}}$$

$$\therefore \left(\frac{W}{L}\right)_1 \approx 30.2$$

7.66

$$
\begin{aligned}
P &= V_{DD} I_{D1} = 1 \text{ mW} \\
I_{D1} &= |I_{D2}| = 556 \text{ µA} \\
V_{ov1} &= V_{GS1} - V_{TH} = \sqrt{2 I_D} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 = 200 \text{ mV} \\
\left(\frac{W}{L}\right)_1 &= \boxed{138.9} \\
A_v &= -\frac{g_{m1}}{g_{m2}} \\
&= -\frac{\sqrt{2 \mu_n C_{ox} \left(\frac{W}{L}\right)_1 I_{D1}}}{\sqrt{2 \mu_n C_{ox} \left(\frac{W}{L}\right)_2 |I_{D2}|}} \\
&= -\sqrt{\frac{\left(\frac{W}{L}\right)_1}{\left(\frac{W}{L}\right)_2}} \\
&= -4 \\
\left(\frac{W}{L}\right)_2 &= \boxed{8.68} \\
V_{IN} &= V_{GS1} = V_{ov1} + V_{TH} = \boxed{0.6 \text{ V}}
\end{aligned}
$$

7.67

$$P = V_{DD} I_D = 3\text{ mW}$$

$$I_D = I_1 = \boxed{1.67\text{ mA}}$$

$$R_{in} = \frac{1}{g_m} = \frac{1}{\sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}} = 50\ \Omega$$

$$\frac{W}{L} = \boxed{600}$$

$$A_v = g_m R_D = \frac{1}{50\ \Omega} R_D = 5$$

$$R_D = \boxed{250\ \Omega}$$

7.68

$$
\begin{aligned}
P &= V_{DD} I_D = 2 \text{ mW} \\
I_D &= 1.11 \text{ mA} \\
V_D &= V_G - V_{TH} + 100 \text{ mV} \\
V_{DD} - I_D R_D &= V_G - V_{TH} + 100 \text{ mV} \\
V_G &= V_{DD} \\
A_v &= g_m R_D = \frac{2I_D}{V_{GS} - V_{TH}} R_D = 4 \\
R_D &= A_v \frac{V_{GS} - V_{TH}}{2I_D} \\
V_{DD} - I_D A_v \frac{V_{GS} - V_{TH}}{2I_D} &= V_{DD} - V_{TH} + 100 \text{ mV} \\
V_{GS} &= 0.55 \text{ V} \\
R_D &= \boxed{270\ \Omega} \\
V_S &= V_{DD} - V_{GS} = I_D R_S \\
R_S &= \boxed{1.125 \text{ k}\Omega} \\
\frac{W}{L} &= \frac{2I_D}{\mu_n C_{ox} \left(V_{GS} - V_{TH}\right)^2} = \boxed{493.8}
\end{aligned}
$$

(69) Power $= 5\,mW$

$\therefore I_{DS1} = \frac{5\times10^{-3}}{1.8} = 2.78\,mA$

$gain\ (A_v) = g_m R_D = 5$

$V_{G1} = V_{OUT} = 1.8 - IR_D$

$V_{S1} = I R_S$

Let $R_S = \frac{10}{g_{m1}}$

$\therefore V_{S1} = \frac{10I}{g_{m1}}$

$\therefore V_{GS1} = 1.8 - IR_D - \frac{10I}{g_{m1}}$

$\because I_{DS} = \frac{1}{2} g_m (V_{GS} - V_{TH})$

$2.78\times10^{-3} = \frac{g_m}{2}\left(1.8 - 2.78\times10^{-3} R_D - \frac{2.78\times10^{-2}}{g_m}\right)$

$= 0.9\,g_m - 1.39\times10^{-3} g_m R_D - 1.39\times10^{-2}$

$\because g_m R_D = A_v = 5$

$2.78\times10^{-3} = 0.9\,g_m - 6.95\times10^{-3} - 1.39\times10^{-2}$

$\therefore g_m \approx 26.3\,mS$

and $R_D = \frac{5}{26.3\times10^{-3}} \approx 190\,\Omega$

$R_S = \frac{10}{26.3\times10^{-3}} = 380\,\Omega$

$\because g_m = \sqrt{2\mu_n C_{ox}(\frac{W}{L}) I_{DS1}} \Rightarrow (\frac{W}{L}) \approx 622$

(70) $\because R_s \approx \frac{10}{g_m}$,

$\therefore R_{in} \approx \frac{1}{g_m} = 50\Omega$

i.e. $g_m = 20\,mS$ //

$$|\text{Gain }(A_v)| = \frac{g_m R_D}{1 + g_m R_S} = 4$$

$$g_m R_D = 4 + 4 g_m R_s$$

$$R_D = \frac{4 + 0.08 R_s}{0.02} = 200 + 4R_s \quad \text{——} (1)$$

$\because R_s I_D + V_{GS} - V_{TH} + 0.25 = 1.8 - I_D R_D$ (given)

and $I_D = \frac{1}{2} g_m (V_{GS} - V_{TH})$

i.e. $V_{GS} - V_{TH} = 100\, I_D$

$\therefore R_s I_D + 100\, I_D + 0.25 = 1.8 - I_D R_D$

From (1):

$$R_s I_D + 100 I_D + 0.25 = 1.8 - 200 I_D - 4 I_D R_s$$

$$5 R_s I_D + 300 I_D = 1.55$$

Set $R_s = \frac{10}{g_m} = 500\,\Omega$

$\therefore 2500 I_D + 300 I_D = 1.55$

$\therefore I_D = 0.554\, mA$ //

$\because I_D = \frac{1}{2} g_m (V_{GS} - V_{TH})$

$$0.554 \times 10^{-3} = \frac{1}{2} \times 20 \times 10^{-3} (V_{GS} - 0.4)$$

$\therefore V_{GS} = 0.455\, V$

To find $(\frac{W}{L})$:

$$g_m = \sqrt{2\left(\frac{W}{L}\right)\mu_n C_{ox}\, I_{DS}}$$

$$\therefore \left(\frac{W}{L}\right) \approx 1805$$

To find R_D:

$$\because \quad R_D = 200 + 4R_S \qquad \text{(from (1))}$$

$$R_D = 2200$$

To find R_1 and R_2,

$$\because \quad R_1 + R_2 = 20\,k\Omega$$

and $V_{GS} = V_G - I_D R_S = 0.455V$

i.e. $V_G = 0.732\,V$

$$V_G = \frac{R_1}{R_1 + R_2} \times V_{DD}$$

$$\therefore R_1 = 8133\,\Omega$$

$$R_2 \approx 11.9\,k\Omega.$$

$\therefore R_1 = 8133\,\Omega$, $R_2 = 11.9\,k\Omega$, $R_D = 2200\,\Omega$ $R_S = 500\,\Omega$

$(\frac{W}{L}) = 1805$ $I_{DS} = 0.554\,mA.$

(71) $R_{in} = R_G = 10\,k\Omega$

Power $= 2mW$

$\therefore I_{DS} = \dfrac{2mW}{1.8V} = 1.11mA$

$A_v = \dfrac{R_S}{\frac{1}{g_m} + R_S} = 0.8$

$\therefore R_S = \dfrac{4}{g_m}$ —— ①.

$\because V_{OUT} = \dfrac{V_{DD}}{2} = I_{DS} R_S$

$I_{DS} R_S = 0.9$ —— ②

$\because V_G = 1.8V$ and $V_S = 0.9$

$\therefore V_{GS} = 0.9V$.

From ②, $\because I_{DS} = 1.11mA$

$R_S = \dfrac{0.9V}{1.11mA} \approx 810\Omega$

From ①, $g_m = \dfrac{4}{810\Omega} \approx 4.94\,mS$.

$\because g_m = \left(\frac{W}{L}\right)(\mu_n C_{ox})(V_{GS} - V_{TH})$

$\frac{W}{L} \approx 49.4$

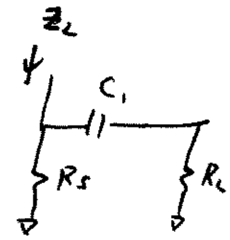
(72) $R_{in} = R_G = 20k\Omega$

$\because$ Power $= 3mW$

$\therefore I_{DS} = \frac{3mW}{1.8V} = 1.67mA$

$V_{x,\text{at DC}} = I_{DS} R_S = 0.9V$

$\therefore R_S = 540\Omega$



Load impedance, $Z_L = R_S // (\frac{1}{sC_1} + R_L)$
(at 100 MHz)

$= 540 // (\frac{1}{2\pi \times 10^8 C_1} + 50)$

Voltage gain $(A_v) = \frac{Z_L}{\frac{1}{g_m} + Z_L}$

$g_m = \frac{2 I_{DS}}{V_{GS} - V_{TH}}$

$= \frac{2 \times 1.67 \times 10^{-3}}{(1.8 - 0.9) - 0.4}$

$= 6.67mS.$

$\therefore A_v = \frac{Z_L}{+ Z_L} = 0.8$

$Z_L = 120 + Z_L(0.8)$

$\therefore Z_L = 150$

$$\therefore 150 = 540 \,//\, \left(\frac{1}{2\pi \times 10^8 C_1} + 50 \right)$$

$$= 540 \,//\, \left[\frac{1 + 50(2\pi \times 10^8 C_1)}{2\pi \times 10^8 C_1} \right]$$

$$= \frac{540 \times \frac{1 + 50(2\pi \times 10^8 C_1)}{2\pi \times 10^8 C_1}}{540 + \frac{1 + 50(2\pi \times 10^8 C_1)}{2\pi \times 10^8 C_1}}$$

$$\therefore \frac{1 + 50(2\pi \times 10^8 C_1)}{2\pi \times 10^8 C_1} \approx 208$$

$$\therefore C_1 \approx 10.1\,pF$$

To find $\left(\frac{W}{L}\right)$:

$$\because g_m = \left(\frac{W}{L}\right) \mu_n C_{ox} (V_{GS} - V_{TH})$$

$$\frac{W}{L} = 66.7$$

$$\therefore \frac{W}{L} = 66.7, \quad C_1 = 10.1\,pF, \quad R_S = 540\,\Omega.$$

7.73

$$
\begin{aligned}
P &= V_{DD} I_{D1} = 3 \text{ mW} \\
I_{D1} &= I_{D2} = 1.67 \text{ mA} \\
A_v &= \frac{r_{o1} \parallel r_{o2}}{\frac{1}{g_{m1}} + r_{o1} \parallel r_{o2}} \\
&= \frac{r_{o1} \parallel r_{o2}}{\frac{1}{\sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right)_1 I_{D1}}} + r_{o1} \parallel r_{o2}} \\
&= 0.9 \\
r_{o1} &= r_{o2} = \frac{1}{\lambda I_{D1}} = 6 \text{ k}\Omega \\
\left(\frac{W}{L}\right)_1 &= \boxed{13.5}
\end{aligned}
$$

Let $V_{ov2} = V_{GS2} - V_{TH} = 0.3$ V. Let's assume that $V_{OUT} = V_{DS2} = V_{ov2}$.

$$
\begin{aligned}
V_{GS2} &= V_b = V_{ov2} + V_{TH} = \boxed{0.7 \text{ V}} \\
\left(\frac{W}{L}\right)_2 &= \frac{2 I_{D2}}{\mu_n C_{ox} \left(V_{GS2} - V_{TH}\right)^2 \left(1 + \lambda V_{DS2}\right)} \\
&= \boxed{161} \\
V_{GS1} &= V_{TH} + \sqrt{\frac{2 I_{D1}}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left(1 + \lambda V_{DS1}\right)}} \\
V_{DS1} &= V_{DD} - V_{DS2} = 1.5 \text{ V} \\
V_{GS1} &= 1.44 \text{ V} \\
V_{IN} &= V_{GS1} + V_{DS2} = \boxed{1.74 \text{ V}}
\end{aligned}
$$